Name: _____

SM3 6.2 Convert Between Degrees & Radians

Vocabulary: Degree: A measurement of an angle representing $\frac{1}{360}$ of a full rotation. We use a superscripted small circle to indicate the degrees unit, °

Radian: The standard unit of angle measurement, the radian is a ratio of the length of an arc divided by the radius of the circle. Because radians are numbers and have no "unit" associated with them, we omit writing "radians" after radian measures.

A circle with a radius of one unit has a circumference of exactly 2π (about 6.283185) units. As $\frac{2\pi}{1} = 2\pi$, it takes 2π radians for a full rotation. We use $360°$ to measure a full rotation. Therefore, these two angular measures must be equal $2\pi = 360°$. It follows that $\pi = 180°$.

We need to be free to convert a measurement from degrees to radians or vice versa. Using $\pi = 180°$, we can take any radian measurement of an angle and substitute $180°$ for a factor of π radians. Conversely, using $\pi = 180°$, we can take any degree measurement of an angle and substitute π for a factor of $180°$.

Example: Convert 4π to degrees.

$$4\pi = 4(180°) = 720°$$

Example: Convert $900°$ to radians.

$$900° = 5(180°) = 5\pi$$

Observe that in the first equation, substituting π for $180°$ has the effect same as dividing by π and multiplying by $180°$, or to keep it to just one operation, multiplying by $\frac{180°}{\pi}$. We can use this ratio of ratios as a conversion factor to move from radians to degrees. We can use its reciprocal, $\frac{\pi}{180°}$, to convert from degrees to radians.

Example: Convert $\frac{\pi}{3}$ to degrees.

$$\frac{\pi}{3} = \frac{\pi}{3}\left(\frac{180°}{\pi}\right) = \frac{180°}{3} = 60°$$

Example: Convert $45°$ to radians.

$$45° = 45°\left(\frac{\pi}{180°}\right) = \frac{45\pi}{180} = \frac{\pi}{4}$$

Memorize: Degrees $\times \frac{\pi}{180°} =$ Radians; Radians $\times \frac{180°}{\pi} =$ Degrees

Practice: Convert each angle measurement as indicated

Convert from degrees to radians:		Convert from radians to degrees:	
$135°$	$270°$	$\frac{5\pi}{6}$	3

Vocabulary: Coterminal Angles: Two angles with the same initial and terminal sides but with different measures.

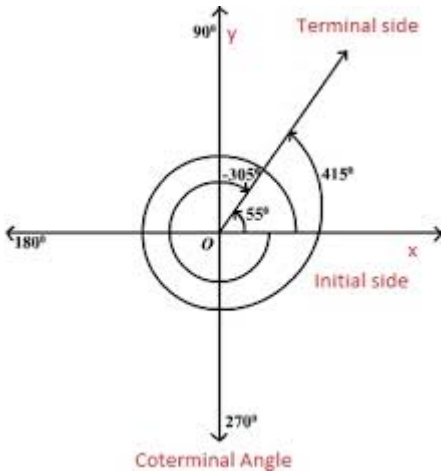
Coterminal angles measured in degrees will always differ by an integer multiple of $360°$.

$55°$ and $415°$ are coterminal angles because their difference is $360°$.

$55°$ and $-305°$ are coterminal angles because their difference is $360°$.

$-305°$ and $415°$ are coterminal angles because their difference is $720°$, which is an integer multiple of $360°$.

Coterminal angles measured in radians will always differ by an integer multiple of 2π.



How many different coterminal angles are there for any given angle? Infinitely many!

Practice: Find a negative coterminal angle and a positive coterminal angle of the given angle.

$135°$	$270°$	$\frac{5\pi}{6}$	$\frac{2\pi}{3}$

As you might guess, we typically do not desire to know the value of all of the coterminal angles. When trying to determine the position of an object, it is normally easiest to use the angle measure that is within a specific domain.

Problems: Convert from degrees to radian measures:

1) $210°$ 2) $80°$ 3) $330°$ 4) $75°$ 5) $45°$ 6) $60°$

Convert from degrees to radians and find a coterminal angle such that $0 \leq \theta < 2\pi$ radians:

7) $450°$ 8) $720°$ 9) $30°$ 10) $-45°$ 11) $-210°$ 12) $-400°$

Problems: Convert from radians to degrees:

13) $\frac{7\pi}{4}$ 14) $\frac{3\pi}{2}$ 15) $\frac{11\pi}{6}$ 16) $\frac{5\pi}{6}$ 17) $\frac{\pi}{9}$ 18) $\frac{\pi}{3}$

Convert from radians to degrees and find a coterminal angle such that $0° \leq \theta < 360°$ radians:

19) 5π 20) $\frac{3\pi}{4}$ 21) $\frac{17\pi}{6}$ 22) $\frac{-\pi}{2}$ 23) 2 24) $\frac{35\pi}{3}$

Tetherball is a game where two opponents hit a ball attached to a pole by a slender nylon rope. The goal of the game is to get the ball to touch the pole. One player wants the ball to travel clockwise until it hits the pole; their opponent wants the ball to travel counterclockwise until it hits the pole. The pole is a 10 foot tall cylinder with a circumference of 6 inches. 8 feet of rope separates the ball from the pole.

When a ball is hit, the radius from pole to hitter's position is the initial side of the angle, the radius from pole to next hitter's position is the terminal side of the angle. You may assume that the players are both skilled enough to move closer or farther from the pole to be in the right position to strike the ball regardless of how much closer the ball is to the pole. To keep the game two-dimensional, you may also assume that the rope is infinitely thin and the rope will always coil on itself rather than slide up or down the pole.

Billy and his sister Joyce set up for a game of tetherball: Joyce is due east of the pole; Billy is due west of the pole and will begin play by serving the ball. Billy wants the ball to travel counterclockwise.

25) Billy bashes the ball counterclockwise! The ball travels a half-way around before Joyce slaps the ball back in the clockwise direction. What is the angle measurement of Billy's serve in radians? What is the angle measurement of Billy's serve in degrees?

26) Joyce's clockwise slap sends the ball over Billy! The ball makes it back to Joyce, who punches it, making it travel even faster! What is the angle measurement of Joyce's slap in radians? What is the angle measurement of Joyce's slap in degrees?

27) The speed of Joyce's punch gives Billy cause to step back so that he can hit the center of the ball accurately. Billy steps clockwise so that he is due northwest of the pole before walloping the ball. What is the angle measurement of Joyce's punch in radians? What is the angle measurement of Joyce's punch in degrees?

28) Billy's wallop accelerates the ball so much that it travels two and a half full rotations around the pole before Joyce steps forward and connects with her elbow. What is the angle measurement of Billy's wallop in radians? What is the angle measurement of Billy's wallop in degrees?

29) Connecting with her elbow, Joyce absorbs most of the ball's energy and it slows down. Billy notices this and adjusts his position to be due southwest from the pole. Unfortunately, he slips on some loose gravel and falls, catching the ball with his face on the way to the ground due southwest from the pole! What is the angle measurement of Joyce's elbow-shot in radians? What is the angle measurement of Joyce's elbow-shot in degrees?

30) Realizing that she's going to be in trouble for hurting her brother, Joyce runs away before her mom can figure out what happened. The ball slowly wraps around the pole completely, resulting in a win for Billy, whose victory is bittersweet because of a minor injury, mainly to his pride. How many times is the rope wrapped around the pole at the moment of victory?